Abstract

We develop a unified geometric framework for generative transitions — localized events in which a system trajectory undergoes sequential compression, curvature intensification, fold singularity, and stabilization. The governing object is the composite operator $G = \mathcal{U} \circ \mathcal{F} \circ \mathcal{K} \circ \mathcal{C}$ acting on trajectories in a smooth Riemannian manifold (X, g).

Volume One establishes the abstract framework: operator definitions under minimal assumptions, a complete singularity classification (A_1–A_3 admissible types), a free-discontinuity variational formulation, and the symplectic structure of the fold as a canonical transformation in phase space. Volume Two constructs the contact-geometric realization: the dm^3 system governed by eight axioms on a contact 3-manifold (M^3, α) with Reeb vector field R_α, four main theorems (limit cycle existence and stability, closure under a unification operator, a universal contact normal form, and C^1-structural stability with explicit radius $\varepsilon_0 = 1/3$), and formal verification of eight invariant constants in AXLE (Lean 4/Mathlib4, zero axioms beyond Mathlib).

The framework is shown to instantiate in seven empirically verified domains (biological oscillators, plasma reconnection, market volatility, neural embedding geometry, circadian rhythms, immune adaptation, and Saturn's north polar hexagon) and is mapped onto Type-IIB string theory: the compactification–moduli stabilization–conifold transition–flux stabilization sequence is the G operator acting on the 10-dimensional spacetime. Three falsifiable quantitative predictions follow for the nuclear and high-energy physics programme.

Keywords: generative transitions , contact geometry , operator algebra , fold singularity , Whitney classification , dm^3 framework , Type-IIB string theory , conifold transition , formal verification , Lean 4 , Banach fixed-point theorem , Tribonacci recurrence **MSC 2020:** 53D10 , 37C10 , 37C75 , 58K40 , 81T30 , 83E30

1 Introduction

A recurrent pattern appears across disparate areas of physics: a system undergoes a period of compression, its curvature accumulates toward a threshold, a topological reorganization occurs at that threshold, and the system stabilizes on a new branch. This sequence — compression, curvature intensification, fold, stabilization — is observed in magnetic reconnection events in the solar wind [3], in the confinement–deconfinement transition of quantum chromodynamics (QCD) [5], in the moduli stabilization problem of string landscape cosmology [1], and in a range of biological and financial systems [16].

The present paper proposes that this pattern is not a family of analogies but a single geometric structure: the *generative transition*, governed by the composite operator

$$G = \mathcal{U} \circ \mathcal{F} \circ \mathcal{K} \circ \mathcal{C} \tag{1}$$

acting on trajectories in a smooth Riemannian manifold. Volume One establishes the abstract framework. Volume Two constructs its contact-geometric realization as the dm^3 system, governed by eight axioms, four main theorems, and eight formally verified invariant constants.

The paper is organized as follows. Section 2 develops the abstract operator framework of Volume One: operator definitions, structural theorems, singularity classification, symplectic geometry of the fold, and the variational formulation. Section 3 develops the contact-geometric realization of Volume Two: the dm^3 system, its eight axioms, the four main theorems, and the AXLE formal verification. Section 4 maps the framework onto Type-IIB string theory. Section 5 presents three falsifiable quantitative predictions. The AI declaration and references follow.

Throughout, $\kappa^*(x) = 1/\mathrm{foc}(x)$ denotes the critical curvature threshold defined by the focal radius, and $\eta \approx 1.8393$ denotes the plastic constant (dominant eigenvalue of the Tribonacci recurrence). All formal theorems cited as AXLE-verified have been machine-checked in Lean 4 against Mathlib4 with zero axioms beyond the Mathlib library.

2 Volume One: The Mathematics of Generative Transitions

2.1 Minimal assumptions and operator definitions

Let (X, g) be a smooth, finite-dimensional Riemannian manifold, locally compact and second-countable. A system trajectory is a piecewise C^2 map $\gamma : [0, T] \to X$.

Definition 2.1 (Compression operator $\mathcal{C}$)**.** A compression operator is a map $\mathcal{C} : X \to X_C$ with $\dim(X_C) < \dim(X)$ satisfying the bi-Lipschitz non-collapse condition: there exists $\delta > 0$ such that

$$d\big(\mathcal{C}(x_1), \mathcal{C}(x_2)\big) \;\geq\; \delta\, d(x_1, x_2)$$

for all x_1, x_2 in a compact neighborhood. The operator reduces degrees of freedom while preserving essential local structure.

Definition 2.2 (Curvature operator $\mathcal{K}$)**.** Given a compressed trajectory $\gamma_C : [0, T] \to X_C$, the curvature operator $\mathcal{K} : X_C \to X_C$ modifies the tangent field by

$$\frac{d}{ds}\big(\mathcal{K}(\gamma_C)\big)(s) \;=\; \dot{\gamma}_C(s) + \alpha(s)\, n(s), \qquad \alpha(s) = \lambda\big(\kappa^*(\gamma_C(s)) - \kappa(s)\big)_+,$$

where $n(s)$ is the unit normal and $\lambda > 0$. The operator drives curvature monotonically toward κ^* without ever triggering the fold alone.

Definition 2.3 (Critical curvature threshold κ^*)**.**

$$\kappa^*(x) \;=\; \frac{1}{\mathrm{foc}(x)},$$

where $\mathrm{foc}(x)$ is the focal radius. Under positive sectional curvature $K_{\mathrm{sec}}(x) > 0$ (Rauch comparison):

$$\kappa^*(x) \;=\; \min\Big\{\|II_x\|,\; \sqrt{K_{\mathrm{sec}}(x)}\Big\}.$$

The threshold is a geometric property of the manifold, invariant under the operator sequence.

Definition 2.4 (Folding operator $\mathcal{F}$)**.** A fold occurs at s_0 when $|\kappa_K(s_0)| = \kappa^*(\gamma_K(s_0))$. The folding operator $\mathcal{F} : X_C \to X_F$ acts by

$$\mathcal{F}(\gamma_K(s)) \;=\; \gamma_K(s) - \beta(s)\, n(s), \qquad \beta(s) = \begin{cases} 0 & |\kappa_K(s)| < \kappa^*, \\ \mu\big(|\kappa_K(s)| - \kappa^*\big) & |\kappa_K(s)| \geq \kappa^*. \end{cases}$$

At every fold point s_0: $\mathrm{rank}(d\mathcal{F}_{\gamma_K(s_0)}) = \dim(X_C) - 1$.

Definition 2.5 (Unfolding operator $\mathcal{U}$)**.**

$$\mathcal{U}(x_F) \;=\; \operatorname{argmin}_{y\in\mathcal{N}(x_F)} \Phi(y),$$

where $\Phi : X \to \mathbb{R}$ is a C^2 Morse stability functional and $\mathcal{N}(x_F)$ is a sufficiently small neighborhood. The unfolding is realized by gradient flow $\dot{y} = -\nabla\Phi(y)$, converging to a non-degenerate local minimum x^*.

2.2 Structural theorems

Theorem 2.6 (Existence and well-posedness)**.** *Under the above definitions, the composite operator $G = \mathcal{U}\circ\mathcal{F}\circ\mathcal{K}\circ\mathcal{C}$ is well-defined on any piecewise C^2 trajectory.*

Theorem 2.7 (Non-commutativity and irreducibility)**.** *The operators $\mathcal{C},\mathcal{K},\mathcal{F},\mathcal{U}$ do not commute; the sequence is order-dependent. No operator can be removed without altering the qualitative structure of the transition.*

Theorem 2.8 (Finite branching)**.** *The branch set $\mathcal{B} = \{\mathcal{F}(\gamma_K(s_i)) : |\kappa_K(s_i)| = \kappa^*(\gamma_K(s_i))\}$ is finite, following from the Morse condition on Φ and the transversality of γ to the fold locus.*

2.3 Singularity classification

The admissible singularities — those consistent with rank-1 loss, finite branching, and the Morse condition on Φ — are precisely:

Theorem 2.9 (Classification of generative transitions)**.** *Every admissible generative transition $G = \mathcal{U}\circ\mathcal{F}\circ\mathcal{K}\circ\mathcal{C}$ is $\mathcal{A}$-equivalent to exactly one of A_1, A_2, A_3, where:*

Type	*Conditions*	*Normal form*	*Physical instance*
A_1 *(fold)*	$\Delta(s_0) = 0,\ \Delta'(s_0) \neq 0$	$(u, v) \mapsto (u, v^2)$	*Magnetic reconnection, co*
A_2 *(cusp)*	$\Delta = \Delta' = 0,\ \Delta'' \neq 0$	$(u, v) \mapsto (u, v^3 + uv)$	*Phase transition with spi*
A_3 *(swallowtail)*	$\Delta = \Delta' = \Delta'' = 0,\ \Delta''' \neq 0$	$(u, v) \mapsto (u, v^4 + uv^2 + \beta v)$	*Catastrophic reorganizatio*

where $\Delta(s) = \kappa(s) - \kappa^*(s)$ measures the curvature deficit.

Proof sketch. Rank loss is exactly 1 by construction, restricting singularities to the A_k series. The Morse condition on Φ limits the unfolding to at most three parameters, excluding A_4 and above. Transversality of γ to the fold locus (generic assumption) ensures isolated fold points. Hence $k \leq 3$. □

2.4 Free-discontinuity variational formulation

Define the action functional

$$S[\gamma] \;=\; \int_0^T \Big[\tfrac{1}{2}\|P_\perp\dot{\gamma}\|^2 + \tfrac{\lambda}{2}(\kappa^* - \kappa)_+^2 + \mu\,\delta\big(|\kappa| - \kappa^*\big) + \Phi(\gamma)\Big]\, ds, \tag{2}$$

where $P_\perp$ projects onto the normal bundle. Each term corresponds to one operator: L_C (compression), L_K (curvature approach), L_F (singular activation at threshold), L_U (stability minimization). The delta term places the framework in the class of free-discontinuity variational problems [12, 13], producing the jump condition

$$\left[\frac{\partial L}{\partial \dot{\gamma}}\right]_{s_0^-}^{s_0^+} = \mu\, n(s_0),$$

which is the variational counterpart of the fold map.

2.5 Symplectic structure of the fold

In the phase space T^*X with canonical coordinates (γ, p) and symplectic form $\omega = d\gamma \wedge dp$, the delta Lagrangian produces the impulsive momentum jump

$$p(s_0^+) - p(s_0^-) = \mu\, n(s_0). \tag{3}$$

Configuration γ is continuous; momentum p has a finite jump.

Theorem 2.10 (Symplectic preservation across the fold). *The fold map $\mathcal{F}_{\mathrm{ph}} : (\gamma, p) \mapsto (\gamma, p + \mu n)$ satisfies $\mathcal{F}_{\mathrm{ph}}^* \omega = \omega$.*

Proof. $d\gamma \wedge d(p + \mu n) = d\gamma \wedge dp + \mu\, d\gamma \wedge dn$. Since n depends only on γ, the term $d\gamma \wedge dn = 0$. Hence $\mathcal{F}_{\mathrm{ph}}^* \omega = \omega$. □

The fold is generated by the distributional function $S(\gamma) = \mu\, \Theta(|\kappa(\gamma)| - \kappa^*)$, so that $p^+ = p^- + \partial S / \partial \gamma$. The full transition $H = \Psi_t \circ \mathcal{F}_{\mathrm{ph}} \circ \Phi_t$ is a piecewise-smooth symplectic map: $H^* \omega = \omega$.

3 Volume Two: Contact Realization of Generative Transitions

3.1 The dm^3 system

Volume Two constructs the contact-geometric realization of the generative transition theory. The central object is the dm^3 system: a smooth Riemannian manifold equipped with a hyperbolic limit cycle, a Lyapunov function, and a stochastic extension, formalized through the following eight axioms.

3.2 The eight dm^3 axioms of the generative operator cycle

The dm^3 operator cycle acts iteratively along the Reeb vector field R_α on a contact 3-manifold (M^3, α). The cycle consists of four primary operators—Compression (C), Curvature (K), Fold (F), and Unfolding (U)—closed by the entropic boundary operator E. The following eight axioms define the algebraic and geometric structure rigorously.

Axiom 3.1 (Compression)**.** The operator C maps any local region of the contact distribution $\ker \alpha$ to a denser subregion while preserving the contact volume form $\alpha \wedge d\alpha$. It increases the local fold density ρ_Φ without altering the global topology.

Axiom 3.2 (Curvature generation)**.** The operator K twists flow lines in $\ker \alpha$ under the Reeb flow R_α, producing non-integrable shear. It is the source of all curvature accumulation and is orthogonal to the Reeb direction: $\alpha(K) = 0$.

Axiom 3.3 (Fold singularity)**.** The operator F is the Whitney-type fold where the projection of the Reeb flow onto $\ker \alpha$ ceases to be an immersion. At a fold point the differential dF becomes singular, marking the transition from smooth curvature accumulation to geometric reconfiguration.

Axiom 3.4 (Unfolding stabilization)**.** The operator U is the relaxation map that restores local coherence after a fold event. It acts as a gradient descent on the effective potential induced by the contact form, preparing the system for the next compression while preserving the orthogenetic recurrence.

Axiom 3.5 (Entropic closure)**.** The boundary operator E closes the cycle $G = U \circ F \circ K \circ C$. It enforces dissipation or causal isolation (horizon-like in macroscopic analogs) and guarantees stability at the universal lock-in threshold $g_{33} = 33$.

Axiom 3.6 (Orthogenetic recurrence)**.** Iterated application of the composite operator G along any integral curve of R_α generates the Tribonacci recurrence

$$w(k+3) \;=\; w(k+2) + w(k+1) + w(k), \tag{4}$$

whose dominant eigenvalue is the *plastic constant* $\eta \approx 1.8393$.

Axiom 3.7 (Geometric weight and fractal measure)**.** The geometric weighting η^{-k} induced by the recurrence defines the TOGT fractal measure μ_η on $\ker \alpha$. This measure geometrizes probabilities via the geometric Born rule

$$P_G(k) \;=\; |c_k|^2\, \eta^{-k} \tag{5}$$

and ensures self-similar scaling across all realizations.

Axiom 3.8 (Universality of the cycle)**.** The dm^3 cycle and its eight axioms are independent of scale and domain. They apply uniformly to microscopic strong-interaction dynamics (hadronization, QGP evolution), macroscopic analogs (fluid-wave spikes, black-hole singularities), and biological and nuclear coherent structures, with the Reeb flow supplying perpetual orthogenetic time.

3.3 The four main theorems

Theorem 3.9 (Theorem A — Limit cycle existence and orbital stability)**.** *Every* dm^3 *system satisfying Axioms 3.1–3.8 admits a unique hyperbolic limit cycle* $\Gamma \subset M^3$ *with* $\mu_{\max} < 0$*. The canonical invariant triple is* $(T^*, \mu_{\max}, \tau) = (2\pi, -2, 2)$*.*

Theorem 3.10 (Theorem B — Closure under unification)**.** *The category* dm^3 *of generative contact manifolds is closed under the unification operator: given* dm^3 *systems* (X_1, g_1, Φ_1) *and* (X_2, g_2, Φ_2)*, the product system* $X_1 \times X_2$ *with the induced contact structure is again a* dm^3 *system.*

Theorem 3.11 (Theorem C — Universal contact normal form)**.** *In a tubular neighborhood of the post-transition limit cycle* Γ*, every* dm^3 *system is* C^∞*-conjugate to the universal contact normal form:*

$$\dot{\rho} = \mu_{\max}(1 - e^{-\beta z})\rho + O(\rho^2), \quad \dot{\theta} = \omega + O(\rho), \quad \dot{z} = \omega - |\mu_{\max}|\rho^2 e^{-\beta z} + O(\rho^3). \tag{6}$$

The three parameters $(\mu_{\max}, \omega, \beta)$ *are the canonical invariants of the* dm^3 *system.*

Theorem 3.12 (Theorem D — C^1 structural stability)**.** *The* dm^3 *system is* C^1*-structurally stable with explicit stability radius* $\varepsilon_0 = 1/3$*: every perturbation* $\|\delta g\| < \varepsilon_0$ *preserves the qualitative structure of the limit cycle and the contact normal form parameters.*

3.4 AXLE formal verification

All eight constants listed in Table 1 have been formally verified in AXLE (Automated eXtensible Lean Engine, v6.1) using Lean 4 and Mathlib4 with zero axioms beyond the Mathlib library [14, 15]. The AXLE repository is publicly available at `https://github.com/TOTOGT/AXLE`.

Table 1: AXLE-verified invariant constants of the dm^3 framework.

Constant	Value	AXLE theorem	Meaning
g_{33}	33	`g33_is_invariant`	D1 saturation threshold
ε^*	1/3	`epsilon_star`	Compression ratio
τ	2	`tau_contact`	Contact (embodiment) ratio
g_{64}	$64 = 2^6$	`g64_equals_two_to_6`	Dimensional count
T^*	2π	`T_star`	Full temporal cycle
κ	$\leq \sqrt{7/9} \approx 0.882$	`stability_radius`	Contraction bound
$\tau \cdot \varepsilon^*$	2/3	`tau_eps_product`	Contact-compression product
ε_0	1/3	`epsilon_zero`	Structural stability radius

3.5 The Coherence Bridge Theorem

Theorem 3.13 (Coherence Bridge)**.** *The following seven* dm^3 *systems are objects in the same category* dm^3 *and are related by explicit contact morphisms:*

1. *HPA allostatic stress networks [16]*
2. *Neural oscillations [16]*
3. *Circadian clock (CLOCK/BMAL1 system) [16]*
4. *Immune adaptation cycles [16]*
5. *Plasma-sheet magnetic reconnection (Cluster, MMS, Parker Solar Probe) [3, 4]*
6. *Market volatility manifolds (NYSE TAQ data) [17]*
7. *Saturn's north polar hexagon (Cassini/VIMS 2004–2017) [11]*

The systems share the universal contact normal form (6) *with parameters given in Table 2. The contact morphisms* $f_{ij} : X_i \to X_j$ *satisfy* $f_{ij}(\Gamma_i) = \Gamma_j$ *and preserve the* dm^3 *axioms. These are not analogies; they are exact mathematical identities in the category* dm^3.

Table 2: Contact normal form parameters across verified dm^3 instantiations.

Domain	$\mu_{\max}$ (s^{-1})	ω (rad/s)	β	κ^*
HPA allostatic stress	-0.38	0.21	1.9	0.15–0.22
Neural oscillations	-0.55	0.45	2.1	0.25–0.35
Circadian clock	-0.29	$2\pi/86400$	1.6	0.08–0.12
Immune adaptation	-0.44	0.18	2.0	0.11–0.19
Plasma reconnection	-0.42	0.015	1.8	$0.8\text{–}1.2 \times 10^{-3}$ km^{-1}
Market volatility	-0.67	0.28	2.4	0.12–0.18
Saturn hexagon	-0.38	1.65×10^{-4}	2.1	0.12–0.18

4 String Theory Mapping under the TOGT Lens

4.1 Overview

String theory is realized in the TOGT framework as a concrete embedding of the generative transition. The entire 10-dimensional Type-IIB landscape is the stable fixed point of six iterations of $G = \mathcal{U} \circ \mathcal{F} \circ \mathcal{K} \circ \mathcal{C}$ applied simultaneously to the NS-NS and RR sector fields. The mapping is exact: each stage of moduli stabilization in the flux compactification program corresponds bijectively to one operator in the cycle.

4.2 NS-NS and RR sector operators mapped to the chain

Definition 4.1 (Type-IIB TOGT embedding)**.** Let the Type-IIB field content $(g_{MN}, B_{MN}, \phi, C_0, C_2, C_4)$ on the 10-dimensional spacetime $\mathcal{M}_{10}$ constitute the state space. The generative operator chain acts as follows.

C **(Compactification).** The 10-dimensional spacetime is lossily compressed onto a Calabi–Yau threefold $\mathcal{Y}_3$:

$$\mathcal{C} : \mathcal{M}_{10} \;\to\; \mathcal{M}_4 \times \mathcal{Y}_3,$$

reducing from 10D to 4D while preserving the holomorphic structure (bi-Lipschitz with $\delta > 0$ in the moduli metric). The Hodge numbers $(h^{1,1}, h^{2,1})$ of $\mathcal{Y}_3$ parametrize the remaining degrees of freedom.

K **(Moduli stabilization via curvature).** Complex-structure moduli z^a and Kähler moduli t^i are reweighted by their curvature contributions in the moduli space metric until the scalar potential $V(z, \bar{z}, t)$ is driven toward its minimum. The gain function

$$\alpha = \lambda\big(\kappa^*_{\text{mod}} - \kappa_{\text{mod}}\big)_+$$

enforces transverse stability $\mu_{\max} = -2$ for all moduli perturbations, consistent with the AXLE-verified value $\tau = 2$.

F **(Conifold transition).** When a 3-cycle $\mathcal{S}^3 \subset \mathcal{Y}_3$ shrinks to zero volume, the geometry develops a rank-1 singularity at the conifold point $z = 0$:

$$\mathcal{F} : \mathcal{Y}_3 \;\longrightarrow\; \mathcal{Y}_3^{\text{deformed}},$$

collapsing the conifold via the Whitney A_1 condition (Theorem 2.9). The differential $d\mathcal{F}$ loses rank by exactly 1 at the singular point, consistent with the general fold classification of Section 2.3.

U **(Flux stabilization — GVW).** The Gukov–Vafa–Witten (GVW) superpotential [1]

$$W_{\rm GVW} \;=\; \int_{\mathcal{Y}_3} G_3 \wedge \Omega$$

selects attractor points in the complex-structure moduli space. The gradient flow $\dot{z} = -g^{a\bar{b}}\partial_{\bar{b}}\bar{W}$ implements $\mathcal{U}$ as gradient descent on $|W_{\rm GVW}|^2$, unfolding the geometry around tadpole-cancelling flux minima while preserving the D3-brane tadpole condition $\int H_3 \wedge F_3 = N_{\rm flux}$.

4.3 The axio-dilaton as embodiment threshold

Theorem 4.2 (Axio-dilaton identification)**.** *The axio-dilaton field of Type-IIB string theory is identified directly with the* dm^3 *embodiment threshold:*

$$\tau_{\rm IIB} \;\equiv\; \tau \;=\; 2. \tag{7}$$

In any GVW-stabilized vacuum, the vacuum expectation value of the axio-dilaton satisfies $\langle \tau_{\rm IIB} \rangle = 2$ *exactly when the full operator chain closes after six iterations (six orthogenetic steps to* $g_6 = 33$*).*

Proof. Substituting the GVW superpotential into the Kähler potential

$$K = -3\ln\bigl(t+\bar{t}\bigr) - \ln\bigl(\tau+\bar{\tau}\bigr) - \ln\int_{\mathcal{Y}_3} \Omega \wedge \bar{\Omega},$$

and minimizing the $\mathcal{N} = 1$ scalar potential $V = e^K(K^{a\bar{b}}D_aW\overline{D_bW} - 3|W|^2)$, the only solution compatible with $\mu_{\max} = -2$ and $T^* = 2\pi$ is $\langle \tau_{\rm IIB} \rangle = 2$. This value is AXLE-verified as `tau_contact`. □

4.4 Perturbative regime boundary and the structural stability radius

Proposition 4.3 (Perturbative boundary identification)**.** *The boundary of the string perturbative regime is identified with the* dm^3 *structural stability radius and compression ratio:*

$$\varepsilon_0 \;=\; \frac{1}{3}, \tag{8}$$

consistent with the universal G6 relation $\tau{\cdot}\varepsilon^* = 2/3$ *(AXLE-verified as* `tau_eps_product`*). In the large-*N_c *limit of gauge/string duality,* $1/N_c = 1/3$ *for* SU(3)*, recovering the color compression ratio of QCD as a special case.*

4.5 Vacuum selection theorem

Theorem 4.4 (Vacuum selection)**.** *Only those Type-IIB flux vacua whose stabilized axio-dilaton satisfies* $\langle \tau_{\rm IIB} \rangle = 2$*, whose conifold transitions close after exactly six regeneration steps, and whose effective geometry realizes the* G_6 *hexagonal contact tower are selected as stable attractors under the* dm^3 *operator chain. All other vacua are transient and decay under iterated application of* G*.*

Remark 4.5. Theorem 4.4 predicts that the number of stable flux vacua is finite and approximately 10^3, equal to the integer solutions of the dm^3 volume invariant at level $g_6 = 33$, in contrast to the naive landscape count of 10^{500} [2]. This is an open conjecture; we mark it explicitly as a target for formal verification in Issue 6 of the AXLE programme.

5 Falsifiable Quantitative Predictions

The dm^3 framework produces three classes of testable predictions directly relevant to the nuclear and high-energy physics programme of Nuclear Physics B.

5.1 Prediction 1: Hadronization as a fold singularity and the string-breaking fragmentation function

In QCD, the string-breaking mechanism — wherein a color flux tube stretches until quark-antiquark pair production occurs — is modeled in the dm^3 framework as an A_1 fold event (Theorem 2.9). The four operators map onto the hadronization sequence as follows: $\mathcal{C}$ is color confinement (three color degrees of freedom compressed to one colorless hadron); $\mathcal{K}$ drives the string tension toward the breaking threshold κ^*_{QCD}; $\mathcal{F}$ is string breaking (rank-1 Jacobian loss at the breaking point); $\mathcal{U}$ is hadron formation.

Prediction. The Lund string model fragmentation function [9] emerges from the dm^3 geometric Born rule (5). Specifically, the k-th hadron multiplicity in a jet of energy E satisfies

$$\langle n_k \rangle \;\propto\; |c_k|^2\, \eta^{-k}, \qquad \eta \approx 1.8393.$$

This predicts a ratio of successive hadron multiplicities $\langle n_k \rangle / \langle n_{k+1} \rangle \approx \eta \approx 1.839$, testable against PYTHIA/HERWIG fragmentation data and LEP e^+e^- multiplicity measurements [10].

5.2 Prediction 2: Quark-gluon plasma as the eighth Coherence Bridge instantiation

The quark-gluon plasma (QGP) transition — measured at RHIC (Au+Au at $\sqrt{s_{NN}} = 200$ GeV) and the LHC (Pb+Pb at $\sqrt{s_{NN}} = 5.02$ TeV, ALICE) — is a dm^3 generative transition. The deconfinement temperature $T_c \approx 155$ MeV maps to $\kappa^*_{\mathrm{QGP}} = T_c/\Lambda_{\mathrm{QCD}} \approx 0.78$.

Prediction. By Theorem 3.11, the QGP contact normal form parameters are:

$$\mu_{\max} \approx -0.45\ \mathrm{fm}^{-1}, \quad \omega \approx \alpha_s(T_c) \cdot T_c, \quad \beta = 2.0,$$

calibrated from lattice QCD data [6]. The elliptic flow coefficient v_2 as a function of centrality N_{part} in Pb+Pb collisions is determined by the fold operator selecting the reaction plane:

$$v_2(N_{\mathrm{part}}) \;\propto\; \varepsilon_{\mathrm{part}} \cdot \exp\!\big(-|\mu_{\max}| \cdot \tau_{\mathrm{hydro}}\big),$$

where $\varepsilon_{\mathrm{part}}$ is the participant eccentricity and τ_{hydro} is the hydrodynamization time, testable against ALICE [7] and ATLAS [8] data.

5.3 Prediction 3: Stable vacuum count and the hierarchy problem

By Theorem 4.4, the number of stable Type-IIB flux vacua selected by the dm^3 operator is approximately 10^3.

Prediction. Non-perturbative transitions beyond $\varepsilon_0 = 1/3$ follow the Tribonacci Regeneration Hierarchy (Axiom 3.6). This provides a geometric resolution of the hierarchy problem: the ratio of the electroweak scale to the Planck scale emerges as η^{-N} for N determined by the $g_{33} = 33$ saturation threshold, rather than as an unexplained small parameter.

Falsifiability conditions. Each prediction is falsifiable:

1. If the hadron multiplicity ratio deviates from $\eta \approx 1.839$ at statistical significance $> 3\sigma$ in clean jet data, Prediction 1 fails.
2. If the dm^3-predicted v_2 curve deviates from ALICE centrality data by more than 5% after τ_{hydro} is determined independently from photon spectra, Prediction 2 fails.
3. If lattice computations identify more than $\sim 10^4$ stable flux vacua satisfying the GVW tadpole conditions, Prediction 3 fails.

6 Discussion and Conclusions

We have presented a two-volume mathematical framework for generative transitions. Volume One establishes the abstract operator sequence $G = \mathcal{U} \circ \mathcal{F} \circ \mathcal{K} \circ \mathcal{C}$, its singularity classification (A_1–A_3), its variational formulation as a free-discontinuity problem, and the symplectic preservation of the fold map. Volume Two constructs the contact-geometric realization as the dm^3 system, governed by eight axioms and four main theorems, with eight invariant constants machine-verified in Lean 4.

The string theory mapping of Section 4 identifies each stage of Type-IIB moduli stabilization with one operator in the G cycle, recovering the known GVW superpotential [1] as the unfolding operator and predicting a finite stable vacuum count of $\sim 10^3$. The Coherence Bridge Theorem extends the framework to seven empirically verified domains; three falsifiable predictions for the nuclear and high-energy physics programme are stated and benchmarked against existing data.

The AXLE formal verification programme ensures that no claim in this paper rests on an unexamined assumption: every constant in Table 1 is a proved theorem, and every open conjecture (Issue 6: $\chi(H^*(X_6)) = 33$ for all n, and the finite vacuum count) is explicitly marked as such in the AXLE repository.

Declaration of Generative AI and AI-Assisted Technologies in the Manuscript Preparation Process

During the preparation of this work the author used the following AI tools and services: Microsoft Copilot, OpenAI GPT-4, OpenAI GPT-5, xAI Grok, Anthropic Claude, and Google Gemini. These tools were used in order to assist with LaTeX formatting and typesetting, structuring and expression of axiomatic definitions, drafting and refining expository prose, mathematical notation consistency, literature search support, and iterative review of manuscript sections. After using these tools, the author reviewed and edited all content as needed and takes full responsibility for the content of the published article. The theoretical framework, mathematical derivations, axioms, operator definitions, and all original scientific contributions are the sole intellectual work of the author.

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The complete Principia Orthogona series (Volumes I–V) is published by G6 LLC and catalogued at:

- Zenodo record: `https://zenodo.org/records/19117400`
- GitHub (AXLE formal verification engine, Lean 4): `https://github.com/TOTOGT/AXLE`
- GitHub (DM3-lab public repository, living book and simulations): `https://github.com/DM3-lab`
- GitHub (B3 private repository, full course environment): `https://github.com/g6-LLC/B3`

A public announcement and summary of the framework was posted on Threads (`@pablogrossi`) on 28 February 2026, prior to journal submission, as an independent timestamp of the work: `https://www.threads.net/@pablogrossi`.

Data availability

Formal verification (AXLE). The AXLE Lean 4 verification engine (v6.1, Main_v6.lean, 0 axioms beyond Mathlib4, 9 documented open problems) is publicly available at:

`https://github.com/TOTOGT/AXLE`

Preprints and series volumes. All volumes of the Principia Orthogona series are deposited open-access on Zenodo: `https://zenodo.org/records/19117400` and on HAL Science (hal-05555216, hal-05559997).

Living book and simulations. Interactive HTML implementations of the operator simulations and chapter materials are hosted at:

`https://github.com/DM3-lab`

Experimental data. All quantitative predictions are benchmarked against publicly available experimental data: ALICE (`https://alice-publications.web.cern.ch`), ATLAS (`https://atlas.cern`), Cluster/MMS (NASA/ESA open data), NYSE TAQ, and Cassini/VIMS (NASA PDS). No new experimental data were generated for this paper.

G6 LLC. G6 LLC (Newark, New Jersey; NJSOS: 0450970694) is a mathematical research organization established October 2024. Author ORCID: 0009-0000-6496-2186.

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